

Determination of quantum-noise parameters of realistic cavities

A.A. Semenov,^{1,2,*} W. Vogel,¹ M. Khanbekyan,³ and D.-G. Welsch³

¹ Institut für Physik, Universität Rostock, Universitätsplatz 3, D-18051 Rostock, Germany

²Institute of Physics, National Academy of Sciences of Ukraine, Prospect Nauky 46, UA-03028 Kiev, Ukraine

³Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena, Max-Wien-Platz 1, D-07743 Jena, Germany

(Dated: February 1, 2008)

A procedure is developed which allows one to measure all the parameters occurring in a complete model [A.A. Semenov et al., Phys. Rev. A **74**, 033803 (2006); quant-ph/0603043] of realistic leaky cavities with unwanted noise. The method is based on the reflection of properly chosen test pulses by the cavity.

PACS numbers: 42.50.Lc, 42.50.Pq, 42.50.Nn, 03.65.Yz

I. INTRODUCTION

Optical cavities play an important role for a variety of experiments in quantum optics [1]. The possibilities of realizing strong coupling between cavity modes and atoms inside the cavities make them of interest for the efficient transfer of quantum states between the radiation field and atoms, as desired in quantum information processing. For describing the coupling of radiation into and out of a cavity, quantum stochastic input-output relations [2] and quantum field theoretical concepts were developed [3].

A realistic description of the input-output behavior of leaky cavities requires a careful consideration of possible losses that may significantly alter the nonclassical properties of light. Cavity parameters, such as the transmission and loss coefficients, the free spectral range (FSR), the cavity-decay rate Γ , and the Q -factor, have been measured by using the direct transmission of light through the cavity [4, 5] and by monitoring nonclassical characteristics of light [6]. It has been demonstrated that in high- Q cavities the losses caused by absorption and scattering may be of the same order of magnitude as the wanted outcoupling of the field [4]. For describing additional effects of unwanted losses associated with absorption and scattering, attempts have been made to introduce additional input-output ports into the quantum noise theory, see e.g. Refs. [7, 8]. It has been shown that even small unwanted losses due to absorption and scattering may substantially diminish nonclassical signatures of the outgoing pulse [7]. Hence for any application that requires a precise transfer of quantum states of light into or out of a cavity, a careful description of the properties of the used cavity is indispensable.

Recently it has been demonstrated that a complete cavity model, describing all the unwanted losses, requires additional noise terms in both the quantum Langevin equation and the input-output relation [9]. This may lead to new effects, such as the superposition of the in-

put field with the outgoing field from the cavity in a common nonmonochromatic mode. In principle, all parameters necessary for the complete description of cavities can be expressed in terms of the radiation and absorption coefficients of the mirrors [10]. In practice, however, the parameters used in such a model will also depend on scattering, mirror alignment, and other conditions of the experiment. Hence, an operational procedure for the determination of all the cavity parameters is desired for a correct descriptions of the quantum effects to be expected.

In the present contribution we propose a method to determine the cavity parameters needed for a complete description of the quantum noise effects of a realistic cavity. It is based on the reflection of light pulses of different lengths by the cavity. The absorption and scattering of pulses with a spectrum that is much wider than the cavity decay rate is shown to depend only on the additional term in the input-output relation. This fact will appear to be useful for the needed measurement procedure.

II. THE CAVITY MODEL

Let us consider the recently proposed model of leaky cavity, which has a partial transmitting mirror, with unwanted noise [9]. The cavity is described by the quantum Langevin equation

$$\dot{\hat{a}}_{\text{cav}} = -[i\omega_{\text{cav}} + \frac{1}{2}\Gamma] \hat{a}_{\text{cav}} + \mathcal{T}^{(c)} \hat{b}_{\text{in}}(t_1) + \hat{C}^{(c)}(t_1), \quad (1)$$

and the input-output relation

$$\hat{b}_{\text{out}}(t_1) = \mathcal{T}^{(o)} \hat{a}_{\text{cav}}(t_1) + \mathcal{R}^{(o)} \hat{b}_{\text{in}}(t_1) + \hat{C}^{(o)}(t_1). \quad (2)$$

Here, \hat{a}_{cav} is the annihilation operator of an intracavity mode, $\hat{b}_{\text{in}}(t_1)$ is the input-field operator, $\hat{b}_{\text{out}}(t_1)$ is the output-field operator, ω_{cav} is the resonance frequency of the cavity, and Γ is the cavity decay rate. The complex numbers $\mathcal{T}^{(o)}$ and $\mathcal{T}^{(c)}$ are the transmission coefficients describing the outcoupling of the internal field and the incoupling of the input field respectively. The complex number $\mathcal{R}^{(o)}$ is the reflection coefficient at the cavity. The operators $\hat{C}^{(c)}(t_1)$ and $\hat{C}^{(o)}(t_1)$ are associated with

*E-mail address: sem@iop.kiev.ua

the unwanted losses and obey the commutation relations

$$[\hat{C}^{(c)}(t_1), \hat{C}^{(c)\dagger}(t_2)] = |\mathcal{A}^{(c)}|^2 \delta(t_1 - t_2), \quad (3)$$

$$[\hat{C}^{(o)}(t_1), \hat{C}^{(o)\dagger}(t_2)] = |\mathcal{A}^{(o)}|^2 \delta(t_1 - t_2), \quad (4)$$

$$[\hat{C}^{(c)}(t_1), \hat{C}^{(o)\dagger}(t_2)] = |\mathcal{A}^{(c)}| |\mathcal{A}^{(o)}| e^{i\kappa} \cos \zeta \delta(t_1 - t_2), \quad (5)$$

where the numbers $|\mathcal{A}^{(c)}|$, $|\mathcal{A}^{(o)}|$, and $e^{i\kappa} \cos \zeta$ are related to the transmission and reflection coefficients through the constraints

$$\Gamma = |\mathcal{A}^{(c)}|^2 + |\mathcal{T}^{(c)}|^2, \quad (6)$$

$$|\mathcal{R}^{(o)}|^2 + |\mathcal{A}^{(o)}|^2 = 1, \quad (7)$$

$$\mathcal{T}^{(o)} + \mathcal{T}^{(c)*} \mathcal{R}^{(o)} + |\mathcal{A}^{(c)}| |\mathcal{A}^{(o)}| e^{i\kappa} \cos \zeta = 0, \quad (8)$$

which follow from the requirements of preserving the commutation rules, for details see Ref. [9].

The set of complex coefficients ω_{cav} , Γ , $\mathcal{T}^{(o)}$, $\mathcal{T}^{(c)}$, and $\mathcal{R}^{(o)}$ completely describe the radiative and the noise properties related to the considered intracavity mode. In view of Eqs. (6-8), these parameters attain only values within a restricted domain [9]. By supposing that the resonance frequency ω_{cav} and the FSR are already known, we will formulate an operational procedure for the measurement of the other parameters of this set. Our procedure is based on measuring the reflection efficiencies of different test pulses by the cavity.

III. REFLECTION OF QUANTUM LIGHT BY THE CAVITY

We start with combining solution of the quantum Langevin equation (1) and the input-output relation (2), also c.f. [9],

$$\begin{aligned} \hat{b}_{\text{out}}(t_1) &= \hat{a}_{\text{cav}}(0) F^*(t_1) + \int_{-\infty}^{+\infty} dt_2 G^*(t_1, t_2) \hat{b}_{\text{in}}(t_2) + \hat{C}(t_1), \end{aligned} \quad (9)$$

where $\hat{a}_{\text{cav}}(0)$ is the operator of the cavity mode at initial time,

$$F^*(t_1) = \mathcal{T}^{(o)} e^{-(i\omega_{\text{cav}} + \frac{\Gamma}{2})t_1} \Theta(t_1), \quad (10)$$

$$G^*(t_1, t_2) = \mathcal{T}^{(c)} \xi^*(t_1, t_2) + \mathcal{R}^{(o)} \delta(t_1 - t_2), \quad (11)$$

$$\xi^*(t_1, t_2) = \mathcal{T}^{(o)} e^{-(i\omega_{\text{cav}} + \frac{\Gamma}{2})(t_1 - t_2)} \Theta(t_1) \Theta(t_1 - t_2). \quad (12)$$

Equation (9) can be rewritten by using two complete orthogonal sets of functions $\{U_n^{\text{in}}(t_1), n = 0, \dots, +\infty\}$ and

$\{U_n^{\text{out}}(t_1), n = 0, \dots, +\infty\}$ associated with input and output fields respectively,

$$\hat{b}_{\text{in(out)}}(t_1) = \sum_{n=0}^{+\infty} U_n^{\text{in(out)}}(t_1) \hat{b}_{\text{in(out)};n}, \quad (13)$$

$$\hat{b}_{\text{in(out)};n} = \int_{-\infty}^{+\infty} dt_1 U_n^{\text{in(out)*}}(t_1) \hat{b}_{\text{in(out)}}(t_1). \quad (14)$$

Suppose that the function $U_0^{\text{out}}(t_1)$ has the form of the pulse extracted from the cavity, i.e., the cavity-associated output mode (CAOM),

$$U_0^{\text{out}}(t_1) = \frac{\sqrt{\Gamma}}{|\mathcal{T}^{(o)}|} F^*(t_1). \quad (15)$$

The function $U_0^{\text{in}}(t_1)$ describes the test pulse (TP) to be reflected by the cavity, leading to the output pulse

$$U^{\text{out}}(t_1) = \int_{-\infty}^{+\infty} dt_2 G^*(t_1, t_2) U_0^{\text{in}}(t_2). \quad (16)$$

The function $U_1^{\text{out}}(t_1)$ is chosen such that the expansion of $U^{\text{out}}(t_1)$ in terms of $U_n^{\text{out}}(t_1)$ consists of two components,

$$U^{\text{out}}(t_1) = \sqrt{\eta_{\text{ref}}} e^{i\mu} U_0^{\text{out}}(t_1) + \sqrt{\eta_{\text{ref}}} U_1^{\text{out}}(t_1), \quad (17)$$

see Fig 1.

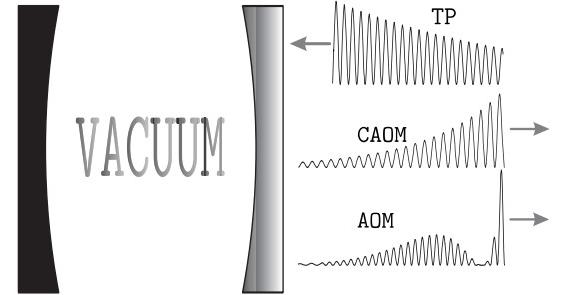


FIG. 1: Reflection of a test pulse (TP) by the cavity. The outgoing field consists of two different nonmonochromatic modes: the cavity-associated output mode (CAOM) and an additional output mode (AOM).

We will refer to the pulse associated with the function $U_1^{\text{out}}(t_1)$ as the additional output mode (AOM). Here η_{ref} and μ are, respectively, the efficiency and the phase of the reflection of the TP into the CAOM. From the orthogonality conditions we obtain

$$\begin{aligned} \sqrt{\eta_{\text{ref}}} e^{i\mu} &= \int_{-\infty}^{+\infty} dt_1 U^{\text{out}}(t_1) U_0^{\text{out}*}(t_1) \\ &= \left[\frac{\mathcal{T}^{(o)} \mathcal{T}^{(c)}}{\Gamma} + \mathcal{R}^{(o)} \right] \\ &\times \frac{\mathcal{T}^{(o)*}}{|\mathcal{T}^{(o)}|} \sqrt{\Gamma} \int_{-\infty}^{+\infty} dt_1 U_0^{\text{in}}(t_1) e^{(i\omega_{\text{cav}} + \frac{\Gamma}{2})t_1}, \end{aligned} \quad (18)$$

where $\bar{\eta}_{\text{ref}}$ is the efficiency of the input-field reflection into the AOM. Using Eq. (17) and the normalization condition for $U_1^{\text{out}}(t_1)$ one derives

$$\bar{\eta}_{\text{ref}} = \int_{-\infty}^{+\infty} dt_1 \left| U^{\text{out}}(t_1) - \sqrt{\underline{\eta}_{\text{ref}}} e^{i\mu} U_0^{\text{out}}(t_1) \right|^2. \quad (19)$$

The total efficiency of the input-field reflection at the cavity is given by the expression

$$\eta_{\text{ref}} = \underline{\eta}_{\text{ref}} + \bar{\eta}_{\text{ref}} = \int_{-\infty}^{+\infty} dt_1 |U^{\text{out}}(t_1)|^2. \quad (20)$$

In the considered representation, Eq. (9) has the form

$$\begin{aligned} \hat{b}_{\text{out};0} &= \sqrt{\eta_{\text{ext}}} \hat{a}_{\text{cav}}(0) + \sqrt{\underline{\eta}_{\text{ref}}} e^{i\mu} \hat{b}_{\text{in};0} \\ &+ \sum_{m=1}^{+\infty} G_{m,0}^* \hat{b}_{\text{in};m} + \hat{C}_0, \end{aligned} \quad (21)$$

$$\hat{b}_{\text{out};1} = \sqrt{\bar{\eta}_{\text{ref}}} \hat{b}_{\text{in};0} + \sum_{m=1}^{+\infty} G_{m,1}^* \hat{b}_{\text{in};m} + \hat{C}_1, \quad (22)$$

$$\hat{b}_{\text{out};n} = \sum_{m=1}^{+\infty} G_{m,n}^* \hat{b}_{\text{in};m} + \hat{C}_n \text{ for } n = 2, 3, \dots, \quad (23)$$

where

$$G_{m,n}^* = \int_{-\infty}^{+\infty} dt_1 dt_2 U_n^{\text{out}*}(t_1) G^*(t_1, t_2) U_m^{\text{in}}(t_2), \quad (24)$$

$$\hat{C}_n = \int_{-\infty}^{+\infty} dt_1 U_n^{\text{out}*}(t_1) \hat{C}(t_1), \quad (25)$$

$$\eta_{\text{ext}} = \frac{|\mathcal{T}^{(o)}|^2}{\Gamma}. \quad (26)$$

Equation (26) defines the efficiency of quantum-state extraction from the cavity [7].

Consider the reflection of the TP, whose form is that of a pulse extracted from another cavity with the same resonance frequency and the cavity decay rate $\Gamma^{(k)}$,

$$U_0^{\text{in}}(t_1) = \sqrt{\Gamma^{(k)}} e^{-\left(i\omega_{\text{cav}} + \frac{\Gamma^{(k)}}{2}\right)t_1} e^{i\varphi^{(k)}} \Theta(t_1). \quad (27)$$

In this case the reflected pulse according to Eq. (16) has the form

$$\begin{aligned} U^{\text{out}}(t_1) &= \sqrt{\Gamma^{(k)}} e^{-\left(i\omega_{\text{cav}} + \frac{\Gamma^{(k)}}{2}\right)t_1} e^{i\varphi^{(k)}} \Theta(t_1) \\ &\times \left\{ \frac{2\mathcal{T}^{(o)}\mathcal{T}^{(c)}}{\Gamma - \Gamma^{(k)}} \left[1 - \exp\left(-\frac{\Gamma - \Gamma^{(k)}}{2}t_1\right) \right] + \mathcal{R}^{(o)} \right\}. \end{aligned} \quad (28)$$

The efficiencies of the reflection into the CAOM and the AOM are, according to Eqs. (18) and (19), given by

$$\underline{\eta}_{\text{ref}}^{(k)} = \frac{4\Gamma\Gamma^{(k)}}{(\Gamma + \Gamma^{(k)})^2} \left| \frac{\mathcal{T}^{(o)}\mathcal{T}^{(c)}}{\Gamma} + \mathcal{R}^{(o)} \right|^2, \quad (29)$$

$$\bar{\eta}_{\text{ref}}^{(k)} = \left(\frac{\Gamma - \Gamma^{(k)}}{\Gamma + \Gamma^{(k)}} \right)^2 \left| \frac{2\mathcal{T}^{(o)}\mathcal{T}^{(c)}}{\Gamma - \Gamma^{(k)}} + \mathcal{R}^{(o)} \right|^2. \quad (30)$$

From Eq. (20) the total efficiency of the reflection, $\eta_{\text{ref}}^{(k)}$, can be found to be

$$\eta_{\text{ref}}^{(k)} = \frac{D}{\Gamma(\Gamma + \Gamma^{(k)})} + \left| \mathcal{R}^{(o)} \right|^2, \quad (31)$$

where

$$D = 4 \left| \mathcal{T}^{(o)} \right|^2 \left| \mathcal{T}^{(c)} \right|^2 + 4\Gamma \operatorname{Re} \left(\mathcal{T}^{(o)} \mathcal{T}^{(c)} \mathcal{R}^{(o)*} \right). \quad (32)$$

This efficiency can be measured, for example, by reflecting the pulse being in a coherent state and comparing the amplitudes before and after reflection.

IV. MEASUREMENT PROCEDURE

For the formulation of the first step of the measurement procedure, we consider the special case $\Gamma \ll \Gamma^{(k)} \ll \text{FSR}$. In this case, as it follows from Eqs. (28) and (31) in the limit $\Gamma^{(k)}/\Gamma \rightarrow +\infty$, the TP does not change its form during reflection (and partial absorption and scattering) according to

$$U^{\text{out}}(t_1) = \mathcal{R}^{(o)} U_0^{\text{in}}(t_1), \quad (33)$$

$$\eta_{\text{ref}} = \left| \mathcal{R}^{(o)} \right|^2. \quad (34)$$

Hence, one can conclude that such pulses do not couple with the intracavity field and the efficiency of their reflection is completely defined by the value of $\mathcal{R}^{(o)}$. This efficiency as well as the corresponding phase shift can be measured. According to Eq. (7) with such kinds of experiments one can check the strength of the noise term $\hat{C}^{(o)}(t)$ in the input-output relation (2). Therefore, by reflecting an appropriate pulse by the cavity one can measure the value of $\mathcal{R}^{(o)}$ and hence of $|\mathcal{A}^{(o)}|$.

The second step of the procedure is the measurement of the two total efficiencies of the reflection $\eta_{\text{ref}}^{(1)}$ and $\eta_{\text{ref}}^{(2)}$ for TP's defined by Eq. (27) with two different parameters $\Gamma^{(1)}$ and $\Gamma^{(2)}$, respectively. One can consider Eq. (31) for $k = 1, 2$ as a system of two algebraic equations for the variables Γ and D . Resolving it we obtain

$$\Gamma = \frac{\Gamma^{(2)} \left(\eta_{\text{ref}}^{(2)} - |\mathcal{R}^{(o)}|^2 \right) - \Gamma^{(1)} \left(\eta_{\text{ref}}^{(1)} - |\mathcal{R}^{(o)}|^2 \right)}{\eta_{\text{ref}}^{(1)} - \eta_{\text{ref}}^{(2)}}, \quad (35)$$

$$D = \Gamma \left(\Gamma + \Gamma^{(k)} \right) \left(\eta_{\text{ref}}^{(k)} - \left| \mathcal{R}^{(o)} \right|^2 \right). \quad (36)$$

Therefore, this part of the measurement procedure allows one to obtain the values of cavity decay rate Γ and of D , Eq. (32), by reflection of different pulses at the cavity.

The third step of the procedure is the measurement of the efficiency of the reflection into the CAOM as given by Eq. (29). Because of the specific form of this expression one can consider, without loss of generality, the efficiency η_{ref} for the case of $\Gamma = \Gamma^{(k)}$. This value can be measured by balanced homodyne detection [11] of the reflected pulse being in the coherent state. The local oscillator field is prepared in the form of the CAOM with an arbitrary phase, see Fig. 2. Such a procedure allows one to measure only the quadrature of the CAOM with suppression of information about the state of the AOM, see discussion of cascaded homodyning in Ref. [9]. Comparing the absolute values for the coherent amplitudes of the input pulse and the CAOM, one can find the corresponding efficiency η_{ref} .

Combining Eq. (29) for $\Gamma = \Gamma^{(k)}$ and Eq. (32), one obtains the expression for $|\mathcal{T}^{(o)}|^2 |\mathcal{T}^{(c)}|^2$,

$$|\mathcal{T}^{(o)}|^2 |\mathcal{T}^{(c)}|^2 = \frac{D}{2} + \Gamma^2 \left(|\mathcal{R}^{(o)}|^2 - \eta_{\text{ref}} \right), \quad (37)$$

and than Eq. (32) can be used for expressing the phase of $\mathcal{T}^{(o)} \mathcal{T}^{(c)}$ as

$$\arg \mathcal{T}^{(o)} \mathcal{T}^{(c)} = \arccos \frac{D - 4|\mathcal{T}^{(o)}|^2 |\mathcal{T}^{(c)}|^2}{4\Gamma |\mathcal{T}^{(o)}| |\mathcal{T}^{(c)}|} - \arg \mathcal{R}^{(o)}. \quad (38)$$

Therefore, this stage of the measurement procedure allows one to obtain the value of $\mathcal{T}^{(o)} \mathcal{T}^{(c)}$.

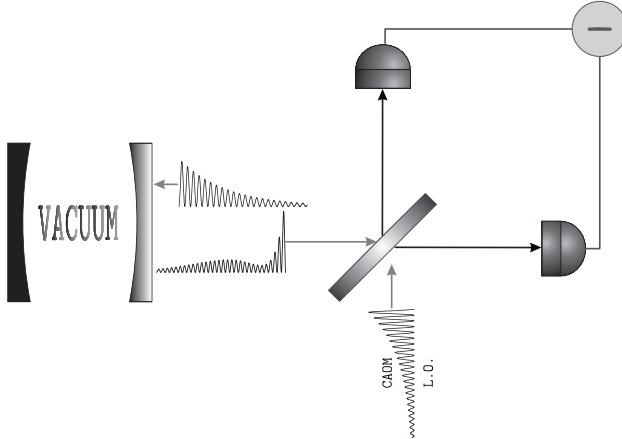


FIG. 2: The scheme of measuring the efficiency η_{ref} of the reflection into the CAOM. The local-oscillator field in the standard scheme of homodyne detection is prepared in the form of the CAOM.

As the fourth step of the measurement procedure one should separate the coefficients $\mathcal{T}^{(o)}$ and $\mathcal{T}^{(c)}$. In the most general case it is impossible to do this by using

only the information about reflection by the cavity. It can be performed directly by measuring the efficiency of the quantum state extraction given by Eq. (26) and the corresponding phase. This kind of experiment allows one, in principle, to obtain the value of $\mathcal{T}^{(o)}$ and evaluate $\mathcal{T}^{(c)}$ using the knowledge of $\mathcal{T}^{(c)} \mathcal{T}^{(o)}$.

As it follows from Eqs. (7) and (8), the difference between the absolute values of $\mathcal{T}^{(o)}$ and $\mathcal{T}^{(c)}$ is caused by deviation of $|\mathcal{R}^{(o)}|^2$ from 1. According to Eqs. (4) and (7), the additional noise term $\hat{C}^{(o)}(t_1)$ in the input-output relation (2) differs from 0 for such cases. This term describes unwanted losses of the input field inside the coupling mirror under reflection by the cavity. From the other hand, for cavities with a negligible value of this noise, the first step of the measurement procedure reveals that $|\mathcal{R}^{(o)}|^2 = 1$, consequently for this case

$$|\mathcal{T}^{(o)}| = |\mathcal{T}^{(c)}| = \sqrt{|\mathcal{T}^{(o)}| |\mathcal{T}^{(c)}|}. \quad (39)$$

For such cavities the absolute values of $\mathcal{T}^{(o)}$ and $\mathcal{T}^{(c)}$ are easily obtained.

V. CONCLUSIONS

Leaky optical cavities with unwanted noise are characterized by several parameters. We have demonstrated that they are uniquely related to the efficiencies and phase shifts of the reflection of different light pulses. The proposed procedure of measuring the cavity parameters is based on the determination of these efficiencies. This can be done by comparing the coherent amplitudes of the incident and reflected light fields. Among others, it has been demonstrated that the specific noise term in the input-output relation can be measured through the reflection of a pulse, whose spectrum is much wider than the cavity-decay rate but narrower than the free spectral range. Altogether, the possibility to measure all the parameters allows one to completely characterize the quantum noise effects of a given nonideal cavity.

Acknowledgements

This work was supported by Deutsche Forschungsgemeinschaft. One of the authors (A.A.S.) gratefully acknowledge the President of Ukraine for financial support. He also thanks A.V. Turchin (Institute of Physics, NAS of Ukraine) and E.E. Mikhailov (College of William and Mary) for useful comments.

- P. Berman (Academic, New York, 1994); H.J. Kimble, Phil. Trans. R. Soc. Lond. A **355**, 2327 (1997); K.J. Vahala, *Nature* **424**, 839 (2003); *The Physics of Quantum Information*, edited by D. Bouwmeester, A. Ekert, and A. Zeilinger (Berlin, Springer, 2000).
- [2] M.J. Collett and C.W. Gardiner, Phys. Rev. A **30**, 1386 (1984); C.W. Gardiner and M.J. Collett, Phys. Rev. A **31**, 3761 (1985).
- [3] L. Knöll, W. Vogel, and D.-G. Welsch, Phys. Rev. A **43** (1991) 543.
- [4] C.J. Hood, H.J. Kimble, and J. Ye, Phys. Rev. A **64**, 033804 (2001).
- [5] M.J. Thorpe, R.J. Jones, K.D. Moll, J. Ye, and R. Lalezari, Optics Express **13**, 882 (2005).
- [6] E. E. Mikhailov, K. Goda, and N. Mavalvala, Phys. Rev. A **74**, 033817 (2006); quant-ph/0605207.
- [7] M. Khanbekyan, L. Knöll, A.A. Semenov, W. Vogel, and D.-G. Welsch, Phys. Rev. A **69**, 043807 (2004).
- [8] C. Viviescas, and G. Hackenbroich, Phys. Rev. A **67**, 013805 (2003); J. Opt. B **6**, 211 (2004).
- [9] A.A. Semenov, D.Yu. Vasylyev, W. Vogel, M. Khanbekyan, and D.-G. Welsch, Phys. Rev. A **74**, 033803 (2006); quant-ph/0603043.
- [10] M. Khanbekyan, L. Knöll, D.-G. Welsch, A.A. Semenov, and W. Vogel, Phys. Rev. A **72**, 053813 (2005).
- [11] H.P. Yuen, and V.W.S. Chan, Opt. Lett **8**, 177 (1982); for a review see D.-G. Welsch, W. Vogel, and T. Opatrný, *Progr. Opt.* **39**, 63 (1999).